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The structure of internal intermittency in turbulent flows at large Reynolds number: experiments on scale similarity

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Some of the statistical characteristics of the breakdown coefficient, defined as the ratio of averages over different spatial regions of positive variables characterizing the fine structure and internal intermittency in high Reynolds number turbulence, have been investigated using experimental data for the streamwise velocity derivative $\partial u/\partial t$ measured in an atmospheric boundary layer. The assumptions and predictions of the hypothesis of scale similarity developed by Novikov and by Gurvich & Yaglom do not adequately describe or predict the statistical characteristics of the breakdown coefficient $q_{r,l}$ of the square of the streamwise velocity derivative. Systematic variations in the measured probability densities and consistent variations in the measured moments show that the assumption that the probability density of the breakdown coefficient is a function only of the scale ratio is not satisfied. The small positive correlation between adjoint values of $q_{r,l}$ and measurements of higher moments indicate that the assumption that the probability densities for adjoint values of $q_{r,l}$ are statistically independent is also not satisfied. The moments of $q_{r,l}$ do not have the simple power-law character that is a consequence of scale similarity.

As the scale ratio l/r changes, the probability density of $q_{r,l}$ evolves from a sharply peaked, highly negatively skewed density for large values of the scale ratio to a very symmetrical distribution when the scale ratio is equal to two, and then to a highly positively skewed density as the scale ratio approaches one. There is a considerable effect of heterogeneity on the values of the higher moments, and a small but measurable effect on the mean value. The moments are roughly symmetrical functions of the displacement of the shorter segment from the centre of the larger one, with a minimum value when the shorter segment is centrally located within the larger one.

1. Introduction

The significance of the ratio of the energy dissipation in a turbulent flow averaged over two different adjoint volumes of different size was first recognized by Yaglom (1966), who employed the concept to derive heuristically certain hypotheses for the structure of the dissipation field given by Kolmogorov (1962) and Obukhov (1962). These hypotheses include the prediction that the probability density of the local turbulent energy dissipation and the dissipation averaged over other appropriate scales should be logarithmically normal. A heuristic argument suggesting that the magnitude of the vorticity in regions of large dissipation will be lognormally distributed has been given by Saffmann (1970).

Gurvich & Yaglom (1967), in an extension of Yaglom's (1966) work, assumed that the analysis could be applied to any non-negative variable characteristic of the fine scales of a turbulent flow, with the resulting prediction that measurable quantities like $(\partial u_i/\partial x_j)^2$ and $(\partial \theta/\partial x_i)^2$, the squared gradients of velocity and scalar fields, respectively, should be distributed lognormally. This question has received a good deal of experimental examination in recent years (by e.g. Gurvich 1966, 1967; Gurvich & Yaglom 1967; Stewart, Wilson & Burling 1970; Gibson, Stegen & Williams 1970; Van Atta & Chen 1970; Wyngaard & Tennekes 1970; Frenkiel & Klebanoff 1971; Kuo & Corrsin 1971). From the data produced by these studies, it appears that the measured probability densities of such non-negative quantities are not generally lognormal, although in some cases appreciable amplitude ranges have been found over which lognormality is a good approximation. Related studies by Kholmyansky (1970), Chen (1971) and Gibson & Masiello (1972) indicate that the probability densities of the spatial averages of such variables can be more nearly lognormal if the averaging length is chosen appropriately.

In an attempt to find more general laws for the structure of the intermittent dissipation field, by employing somewhat less restrictive assumptions than those which lead to lognormality of all non-negative variables, Novikov (1969, 1971) extended Gurvich & Yaglom's (1967) ideas to investigate and predict some of the statistical properties of the breakdown coefficient, which is the ratio of the averages of a non-negative quantity over two volumes or averaging lengths of different size. Novikov obtained, under certain assumptions, universal laws, independent of the large scale of the turbulent field, which are applicable to the statistical characteristics of the breakdown coefficients. Novikov noted that, at that time, available experimental data on intermittency related only to spectra, probability densities, etc., of the values of the averaged or unaveraged non-negative variables, but not to the breakdown coefficients themselves, for which one may expect universal laws under much less restrictive assumptions. The purpose of the present work is to investigate some of the statistical characteristics of the breakdown coefficients, using experimental data for the time derivative of the streamwise velocity $\partial u/\partial t$ obtained in a high Reynolds number atmospheric boundary layer, and to discuss and compare the results obtained with respect to Novikov's theoretical predictions.

The atmospheric data employed, which were very kindly furnished to us in analog form by Dr J. C. Wyngaard of the Air Force Cambridge Research Laboratories, were the same as those used in a previous study of Wyngaard & Pao (1972), in which certain quantities characteristic of the unaveraged fine structure of the turbulence, including skewness, kurtosis and power spectra of the velocity derivative or its square, were calculated and their behaviour compared with the predictions of the refined Kolmogorov theory as modified by Kolmogorov (1962), Obukhov (1962), Yaglom (1966), Gurvich & Yaglom (1967) and Novikov (1965). The persent work thus directly complements and extends the results of this earlier study.

2. Theoretical relations

Similarity concepts have proved fairly useful in dealing with the simplest statistical properties of fluid turbulence. If the set of definitive parameters upon which the statistical characteristics of the quantity under investigation is assumed known, then important relationships, such as those derived for spectra and structure functions by Kolmogorov (1941) and Obukhov (1941), can be obtained. However, it is not always possible to guess the definitive parameters *a priori*, and these parameters may differ for different statistical characteristics. In some problems, it is natural to assume that the statistical characteristics of the field have a definite similarity when the transition is made from one set of scales to another, provided that it is expected that a similar interaction mechanism is dominant throughout the range of scales of interest. This type of similarity, in contrast to similarity with respect to parameters, Novikov calls scale similarity. Here, we shall briefly describe the theoretical results of Novikov (1969, 1971) to be compared with our data in § 4.

Novikov considers a non-negative random function $y(x)$ (in our case the square of the spatial derivative of the streamwise velocity $(\partial u/\partial x)^2$) that is statistically homogeneous and isotropic for spatial length scales less than a certain external scale L. A one-dimensional process of this type is investigated for ease of comparison with experimental work, in which it is common practice to deal mainly with one-dimensional characteristics of the random field. Novikov singles out three segments along x inserted in one another with the lengths $r < \rho < l$, and considers the ratio of the values of the functions $y(x)$ averaged over these segments. This ratio is called the breakdown coefficient $q_{r,l}$, where

$$q_{r,l}(h, x) = y_r(x')/y_l(x) \quad (r < l), \tag{1}$$

$$y_l(x) = \frac{1}{l}\int_{x-\frac{1}{2}l}^{x+\frac{1}{2}l} y(x_1)\, dx_1, \quad -\tfrac{1}{2} \leqslant h = \frac{x'-x}{l-r} \leqslant \tfrac{1}{2}.$$

The inequality for h means that the smaller segment is included in the larger one.

The probability densities of the $q_{r,l}$ for a homogeneous field $y(x)$ depend upon l and r, and, in general, upon h, since the joint probability density for $y_r(x')$ and $y_l(x)$ and therefore the correlation between these two quantities may depend on h. The moments of the $q_{r,l}$ are defined as

$$a_p(r, l, h) \equiv \langle q^p_{r,l}(h, x)\rangle. \tag{2}$$

The dependence on h defines the non-homogeneity of the breakdown. Novikov suggests that it would be desirable to investigate experimentally the non-homogeneity of breakdown, first of all the dependence upon h of the mean value of the breakdown coefficient.

As an example of this non-homogeneity, Novikov considered a first-order

Markov process of non-negative quantities $y(k)$ $(k, \ldots, 1, 2, 3, \ldots)$ with probability density $W(y)$ and transition probability

$$P(y(k+1)|\,y(k)) = \alpha\delta(y(k+1)-y(k))+(1-\alpha)\,W(y(k+1)) \quad (0<\alpha<1).$$

For the case of $r = 1$ and $l = 3$, one has

$$q_{1,3}(h,k) = \frac{3y(k+2h)}{y(k-1)+y(k)+y(k+1)},$$

and

$$a_p(1,3,h) = \langle q_{1,3}^p(h,k)\rangle,$$

where $h = -\frac{1}{2}, 0$, or $\frac{1}{2}$. Novikov shows that $a_1(1,3,h)$ is an even function of h with a minimum value for $h = 0$, and that the degree of non-homogeneity defined by $a_1(1,3,\frac{1}{2})-a_1(1,3,0)$ does not exceed $\frac{3}{16}$ for arbitrary $W(y)$. More generally, for higher moments we find that

$$0 \leqslant a_p(1,3,\tfrac{1}{2})-a_p(1,3,0) \leqslant 3^p(2^p-1)/2^{p+3}.$$

Novikov defines conditions for scale similarity of the $q_{r,l}$ in the interval of scales $L \gg l > r \gg l_*$, where l_* is a microscale defined by the molecular diffusion coefficient. According to Novikov & Stewart (1964), l_* may differ from the usual Kolmorogov microscale $\eta = (\nu^3/\epsilon)^{\frac{1}{4}}$ (where ϵ is the average energy dissipation rate per unit mass of fluid and ν is the kinematic viscosity) by some power of the Reynolds number. The two conditions are that (i) the probability density of $q_{r,l}$ depends only upon the scale ratio l/r and h, and (ii) two sequential breakdown coefficients $q_{r,\rho}$ and $q_{\rho,l}$ having the same h are statistically independent. From these conditions and (2) it follows that all moments of the breakdown coefficient

$$a_p(l/r,h) = \langle q_{r,l}^p(h,x)\rangle \tag{3}$$

must have a power-law variation with l/r:

$$a_p(l/r,h) = (l/r)^{\mu_p(h)}, \tag{4}$$

where

$$\mu_p(h)-\mu_q(h) \leqslant p-q,$$

and

$$\mu_p(h) \leqslant p \quad \text{and} \quad \mu_0(h) \equiv 0. \tag{5}$$

If the non-homogeneity of breakdown (dependence on h) is disregarded, then

$$\mu_p \leqslant \mu+p-2 \quad (p \geqslant 2), \tag{6}$$

$$\mu_1 = 0, \quad 0 < \mu_2 \equiv \mu < 1,$$

where μ is the coefficient introduced by Kolmogorov (1962).

Novikov also finds, using (5), that the requirements of scale similarity are not only sufficient but also necessary in order that the moments have a power-law dependence on l/r, and that (5) also ensures fulfillment of the Carleman condition (see e.g. Prohorov & Rozanov 1967), which is sufficient for the probability distribution of $q_{r,l}$ to be uniquely determined by its moments $a_p(l/r,h)$. Furthermore, he shows that the breakdown coefficient has a lognormal probability distribution in the limiting case of $\ln(l/r) \rightarrow \infty$, a result which appears consistent with the hypothesis of Gurvich & Yaglom (1967) that the probability density of the unaveraged square of variables characteristic of the fine scales of the turbulence is lognormal.

Using a superscript asterisk to indicate quantities related to this limiting lognormal distribution only, the corresponding values of μ_p are

$$\mu_p^* = 1/2p[(p-1)(\mu_2^* - 2\mu_1) + 2\mu_1^*], \tag{7}$$

where $$2\mu_1^* \ln(l/r) = 2\kappa_1 + \kappa_2, \quad \mu_2^* \ln(1/r) = 2(\kappa_1 + \kappa_2), \tag{8}$$

and κ_1 and κ_2 are the mean value and variance of the logarithm of the breakdown coefficient. Since $\kappa_2 > 0$, it follows from (8) that $\mu_2^* - 2\mu_1^* > 0$. As shown by Novikov, the quadratic dependence of the exponent μ_p upon the order p of the moment contradicts condition (5), at least for sufficiently large p. This means that, although the distribution of the subdivision coefficient tends toward the lognormal distribution, the moments do not tend towards the expressions which result from the limiting distribution. Thus, the true distribution is uniquely determined by its moments, but these moments cannot be calculated on the basis of the limiting lognormal distribution, which is not uniquely determined by its moments. According to Orszag (1970), this non-uniqueness renders the lognormal distribution incompatible with deductive theories of turbulence based on moment equations.

The conditions of scale similarity are formally very similar to the assumptions employed by Gurvich & Yaglom (1967) in a heutristic development of the lognormal probability density for the energy dissipation ϵ. If we let $y = \epsilon$, and $l/r = v^{\frac{1}{3}}$, where v is a volume ratio instead of a length ratio, then $q_{r,l}$ is interpreted as the ratio of the energy dissipation averaged over two volumes of ratio v. Yaglom imagined this subdivision to be carried out j times, and assumed that (i) the probability densities of the $q_{r,l}$ were mutually independent for different values of j, and (ii) the probability density of $q_{r,l}$ was the same for all j and depended only on l/r. Yaglom & Gurvich then employed the conditional probability density $P(y_r|y_l)$ to predict that the probability density of y_n for sufficiently large j is lognormal, independent of the value of n. Novikov's analysis does not employ the arguments requiring the conditional probability density and is not concerned with the probability density of y_n, but only with that of the ratio $q_{r,l}$. No specific form is predicted by Novikov for this probability density for arbitrary values of l/r, except in the limiting case of $l/r \to \infty$.

3. Data acquisition and analysis

The data were obtained from a modest hot-wire experiment carried out during the much more extensive atmospheric surface-layer measurements programme described by Haugen *et al.* (1971). Single hot wires, 5 microns in diameter and 1·2 mm long, were operated in the linearized constant-temperature mode (DISA 55D05-15 units) at heights $z = 5{\cdot}66$ m and $11{\cdot}3$ m above a horizontally homogeneous Kansas prairie. The mean velocity U was $3{\cdot}78\,\mathrm{m\,s^{-1}}$ at $z = 5{\cdot}66$ m and $4{\cdot}47\,\mathrm{m\,s^{-1}}$ at $z = 11{\cdot}3$ m. The Kolmogorov microscale η was 0·08 cm at $z = 5{\cdot}66$ m and 0·087 cm at $z = 11{\cdot}3$ m. Streamwise velocity derivatives $\partial u/\partial t$ were obtained

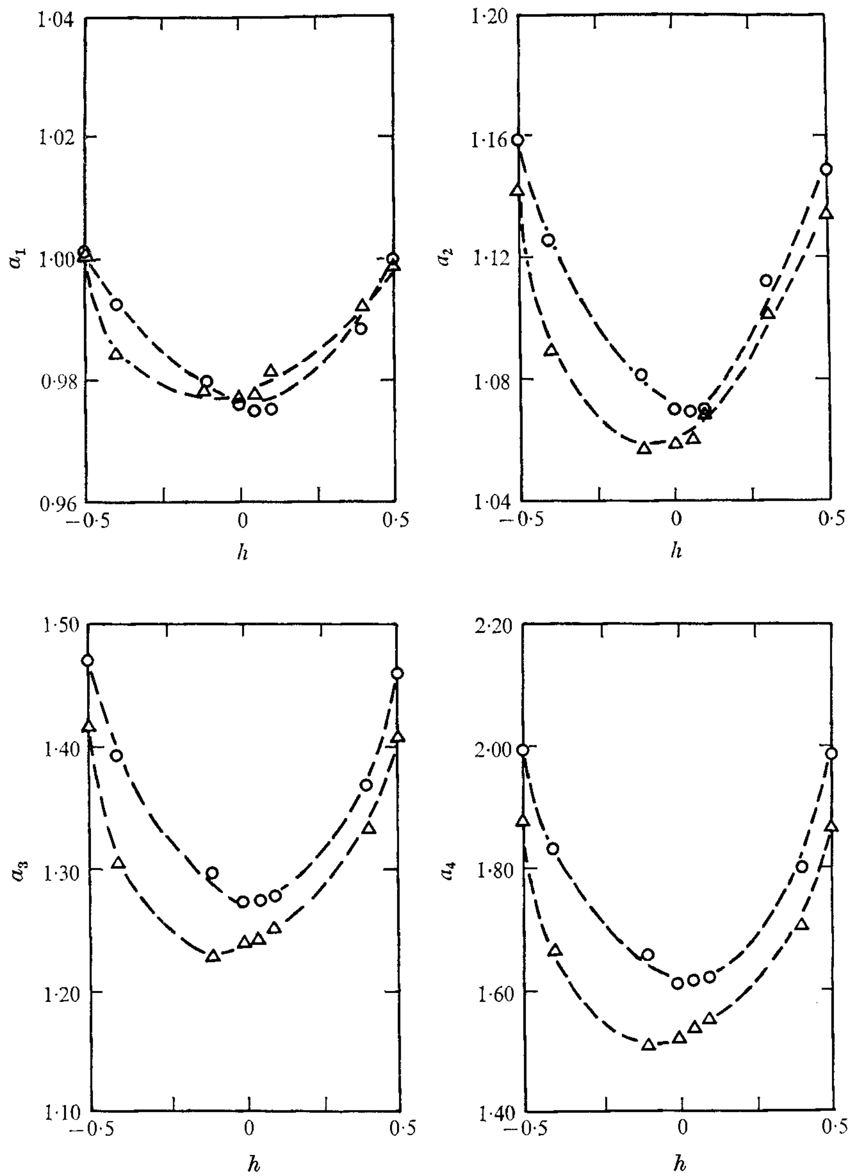

FIGURE 1(a). For legend see facing page.

from four-pole Butterworth filters (Wyngaard & Lumley 1967), which differentiate and low-pass filter (24 db per octave) to eliminate high-frequency noise.

The previous studies of Haugen *et al.* (1971), Wyngaard & Pao (1972) and Tennekes & Wyngaard (1972) provide further details of the data acquisition and estimates of the accuracy of the data. With the aid of Taylor's (1938) hypothesis of a 'frozen' turbulent structure, the temporal fluctuations were interpreted as convected streamwise spatial variations (i.e. $\partial u/\partial t = -U\,\partial u/\partial x$). By assuming that velocities and velocity derivatives are uncorrelated and that

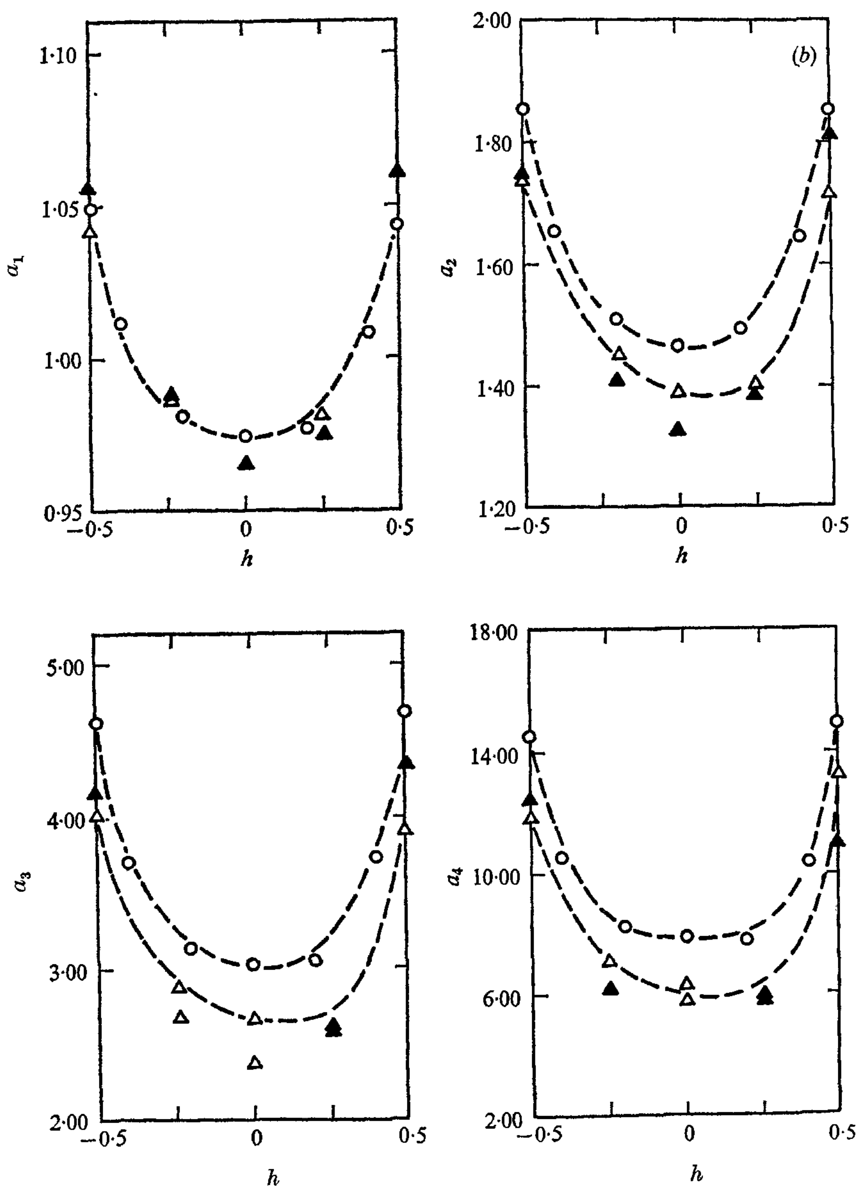

FIGURE 1. (*a*) Dependence of a_p, moments of $q_{r,l}$, on h, relative position of r and l. $l/r = 2$. ○, $l/\eta = 290$; △, 580, $z = 5{\cdot}66$ m. (*b*) Dependence of a_p on h. $l/r = 8$. ○, $l/\eta = 580$; △, 2320, $z = 5{\cdot}66$ m; ▲, 2522, $z = 11{\cdot}3$ m.

the various velocity derivatives are related as in isotropic turbulence, Hekestad (1965) found that for large Reynolds numbers

$$\langle(\partial u/\partial t)^2\rangle = U^2\langle(\partial u/\partial x)^2\rangle\,(1+\langle u^2\rangle/U^2+2\langle v^2\rangle/U^2+2\langle w^2\rangle/U^2).$$

For the velocity fluctuation levels during these runs

$$(\langle u^2\rangle \simeq \langle v^2\rangle+\langle w^2\rangle \simeq 0{\cdot}025U^2)$$

the difference between $\langle(\partial u/\partial t)^2\rangle$ and $\langle(\partial u/\partial x)^2\rangle$ is of the order of 7·5 %. However, since only ratios of averaged squared derivatives were used in the final computations, the correction term inside the brackets ( ) essentially cancels out, so that the shortcomings of Taylor's hypothesis can probably be neglected for the present analysis. According to the results of Wyngaard & Pao (1972), the dissipation overestimate due to the temperature fluctuations was about $2 \sim 3$ %. Calculated spectral attenuation due to wire length was about 10 % at $k\eta = 1{\cdot}5$, where the spectral contribution to $\langle(\partial u/\partial t)\rangle$ is negligibly small, and the data need no correction on this account. These analog data were then later played back in the laboratory at the Department of Applied Mechanics and Engineering Sciences, UCSD, and sampled with a 12 bit analog to digital converter at a sample rate of 4172 samples per second. The digital data were then processed at UCSD with a CDC 3600 computer. The digital data, which were proportional to the time derivative of the longitudinal fluctuating component of the turbulent velocity, were squared, averaged over the appropriate number and group of samples corresponding to various values of l, ρ, r and h. The breakdown coefficients $q_{r,l}$, the ratios of the averaged quantities, are independent of the calibration constant relating the digital data to $\partial u/\partial t$. The new time series for the $q_{r,l}$ were then used to compute the probability densities of the $q_{r,l}$, the moments $a_p(l/r, h)$, and other statistical quantities. Final results were based on 819 200 data samples, except for the largest averaging lengths ($l/\eta \geqslant 2\,320$), for which twice this number of samples were used.

4. Results and discussion

4.1. *The effects of heterogeneity*

The influence of variation of the parameter h, or the effect of heterogeneity, was first examined for the present data by calculating the values of the moments $a_p(l/r, h)$ for fixed values of l/r and for various values of h and l. The value $h = 0$ corresponds to the case when the smaller averaging segment is centrally located within the larger segment, while the limiting cases $h = \pm\frac{1}{2}$ correspond to choosing the smaller segment to lie at the ends of the larger segment. Typical results are shown in figure 1. For $l/r = 2$, the total variation of a_1, the mean value of $q_{r,l}$, over the full range of $h(-\frac{1}{2} \leqslant h \leqslant +\frac{1}{2})$ is only 2–3 %. The relative influence of heterogeneity increases as the order of the moment increases. For $z = 5{\cdot}66$ m, the total variations of a_2, a_3 and a_4 over the full range of h are about 7 %, 14 % and 22 %, respectively. For all moments, the degree of heterogeneity is less than the maximum value for the first-order Markov process given in § 2. All moments have a minimum near $h = 0$ and the variation around $h = 0$ is fairly symmetrical. Thus, qualitatively, the effect of heterogeneity is like that given by the first-order Markov process model used as an example by Novikov. As shown in figure 1, these effects are qualitatively the same for other values of l/r and other vertical heights above the ground. Because of the relatively strong dependence of the higher moments on the value of h, it became clear that any systematic comparison with the theory, which neglects the effect of varying h, would have to be done for

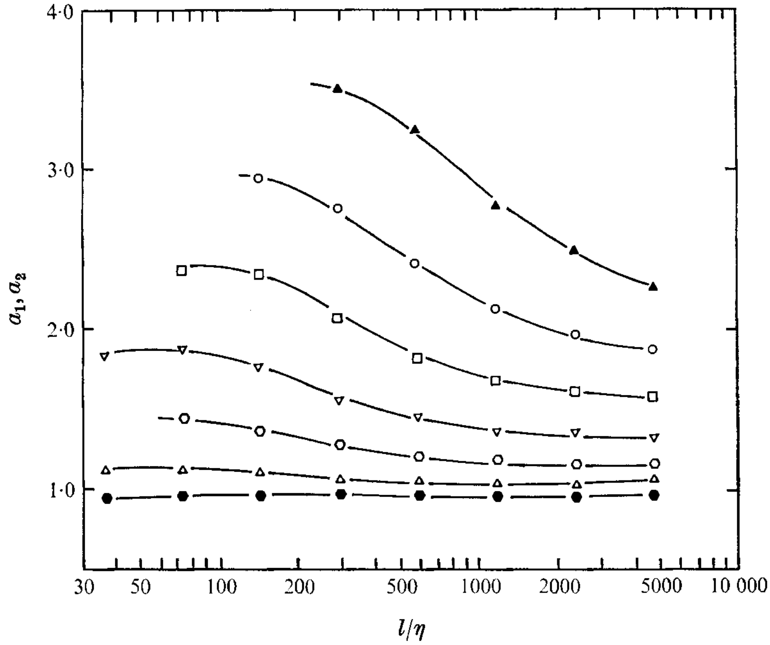

FIGURE 2. First and second moments, a_1 and a_2, as a function of l for fixed values of l/r. $h = 0$. All data in figures 2–15 are for $z = 5 \cdot 66$ m, $h = 0$, unless otherwise specified. a_1: ●, $2 \leqslant l/r \leqslant 64$. a_2: △, $l/r = 2$; ⬡, 4; ▽, 8; □, 16; ○, 32; ▲, 64.

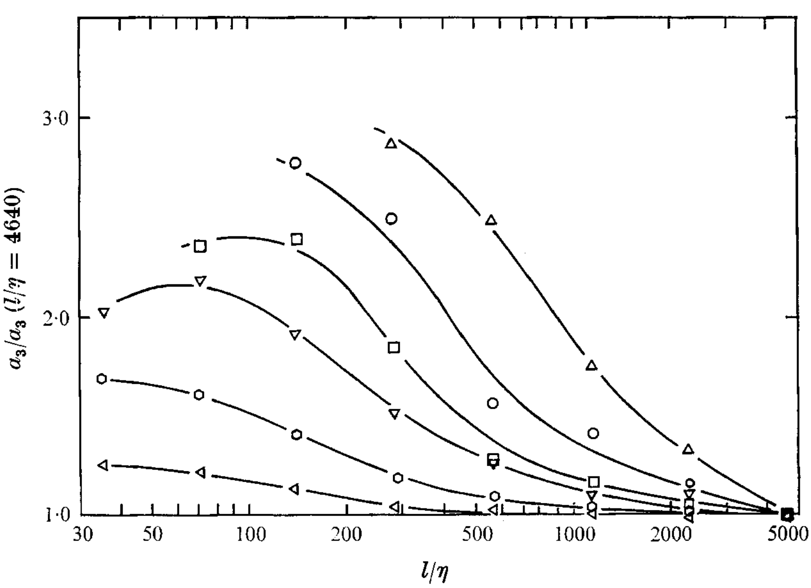

FIGURE 3. Normalized third moment $a_3/a_3\,(l/\eta = 4640)$ as a function of l for fixed values of l/r. ◁, $l/r = 2$, $a_3\,(l/\eta = 4640) = 1 \cdot 20$; ⬡, 4, 1·67; ▽, 8, 2·38; □, 16, 3·83; ○, 32, 5·82; △, 64, 9·39.

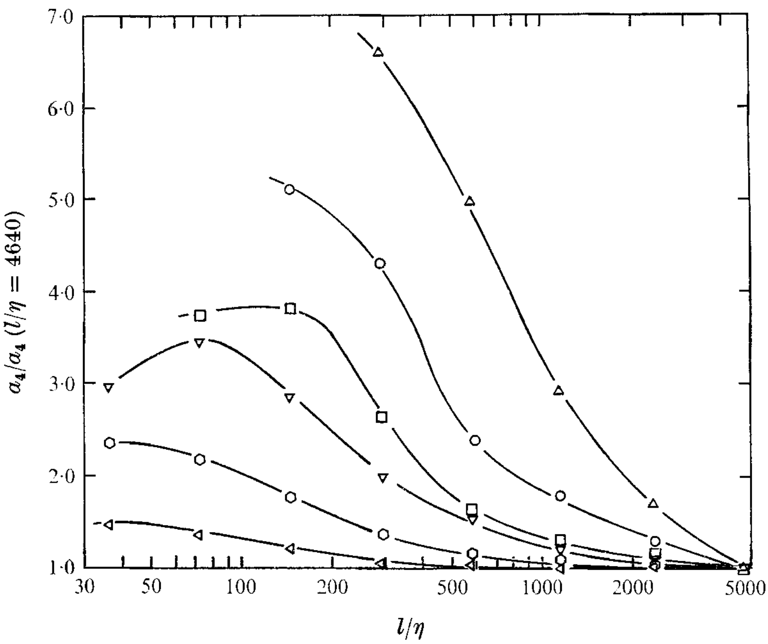

FIGURE 4. Normalized fourth moment $a_4(l)/a_4\ (l/\eta = 4640)$ as a function of l for fixed values of l/r. ◁, $l/r = 2$, $a_4\ (l/\eta = 4640) = 1{\cdot}50$; ⬡, 4, 2·67; ▽, 8, 5·16; □, 16, 12·63; ○, 32, 26·0; △, 64, 58·4.

fixed values of h. Since the rate of variation with h was a minimum at or near $h = 0$, most further detailed calculations were done with the value of h set equal to zero. All data in figures 2–15 are for $h = 0$ and $z = 5{\cdot}66$ m unless specified otherwise.

4.2. *Comparison with the hypotheses and consequences of scale similarity*

The first condition of scale similarity requires that the probability density of $q_{r,l}$ depend only upon the scale ratio l/r. Consequently, all moments would also depend only upon l/r, independent of l in the region of scale similarity. To test this consequence for the data, the moments a_p up to fourth order were computed for $h = 0$ and for various values of l/r. These results are plotted against l in figures 2–4. As shown in figure 2, a_1 is very nearly constant (varying by only about 1 %) for a large range of l in which the scale similarity theory is applicable (say for $5\,\text{cm} \leqslant l \leqslant 3\,\text{m}$ if L is chosen as one-half the vertical height z, an upper limit for locally isotropic scales suggested by the measurements of Van Atta & Chen (1970)), and for the entire range of l/r (from two to sixty-four). The value of a_1 is nearly independent of the value of l/r. Thus from this simple test, one would be tempted to expect that the first condition of scale similarity was satisfied. However, the degree of dependence of the a_p on l increases as p increases. For l/r in the range from two to four, the total variation of a_2 is only about 5–10 % over the same range of l. The small variation of a_2 suggests that to this order a region of scale similarity should be expected. However, for $l/r = 64$,

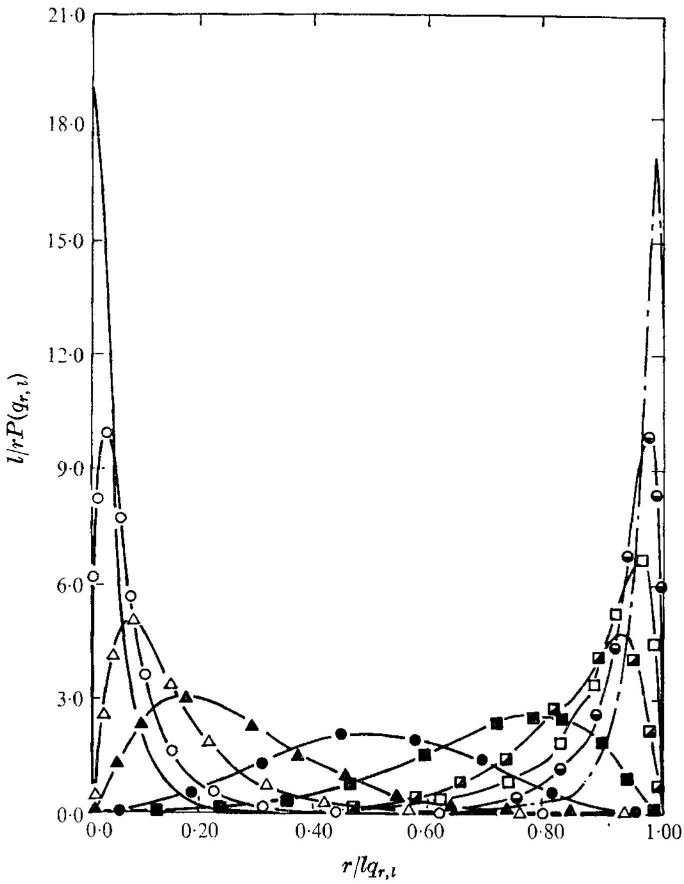

FIGURE 5. Probability densities of $q_{r,l}$ for various values of r/l. $l/\eta = 290$. —, $l/r = 32$; ○, 16; △, 8; ▲, 4; ●, 2; ■, 1·4066; ◪, 1·185; □, 1·123; ◒, 1·076; —·—, 1·0407.

the variation is about 30 % and no restricted range of scale similarity is suggested by the data. The ranges of variation in a_3 and a_4 become progressively larger, approaching an order of magnitude over the range considered for the extreme case of $l/r = 64$. The latter data have been normalized with their values at the largest l, to present them with sufficient resolution in a single plot. There is a tendency for the a_p for $p \geqslant 2$ to become less dependent on l for the smaller values of l, and there is some suggestion in the data that for intermediate values of l/r maximum values may be reached for values of l/η between 70 and 10^2, with the maxima shifting to larger values of l as l/r increases. This range was not completely covered in the computations for the larger values of l/r because the small number of samples occurring within the length r with the present sampling rate made such computations less reliable. However, the apparent maxima which do occur are close to the lower end of the range of possible scale similarity, while the moments change most quickly near the middle of the range of possible scale similarity.

These results clearly indicate that the first hypothesis of scale similarity is

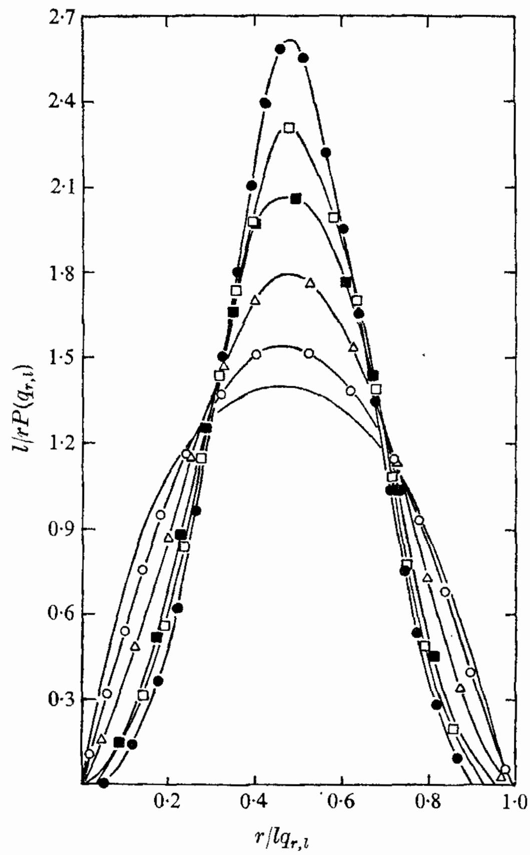

FIGURE 6. Probability densities of $q_{r,l}$ for various values of l for fixed $l/r = 2$. —, $l/\eta = 36\cdot2$; ○, 72·5; △, 145; ■, 290; □, 580; ●, 1160.

not satisfied for our data, i.e. the probability density of $q_{r,l}$ is not a function only of the single variable l/r. To investigate this point in detail, the probability densities for a number of values of r, ρ and l were computed and compared.

The shape of the probability density of $q_{r,l}$ was found to be strongly dependent on the value of l/r. Typical data in figure 5 show that for large l/r the density is sharply peaked near the origin, while for decreasing values of l/r the peak moves to larger values of $q_{r,l}$ and decreases rapidly in size. The probability density becomes nearly symmetrical and the peak reaches its minimum value for $l/r = 2$. As $l/r \to 1$, the probability density approaches a one-sided delta function at $r/lq_{r,l} = 1$, as it does at the origin for $l/r \to \infty$. We note that the large range of l/r is de-emphasized in the figures by normalization of the ordinate with l/r. Similar behaviour was found for other values of l.

For the value of $l/r = 2$, the probability density was very symmetrical for all values of l, as shown in figure 6. As l increased there was a systematic evolution of the density from a flattened distribution with negative curvature everywhere to a more sharply peaked truncated Gaussian-like distribution with positive curvature in the tails. This symmetrical behaviour of the density was found only

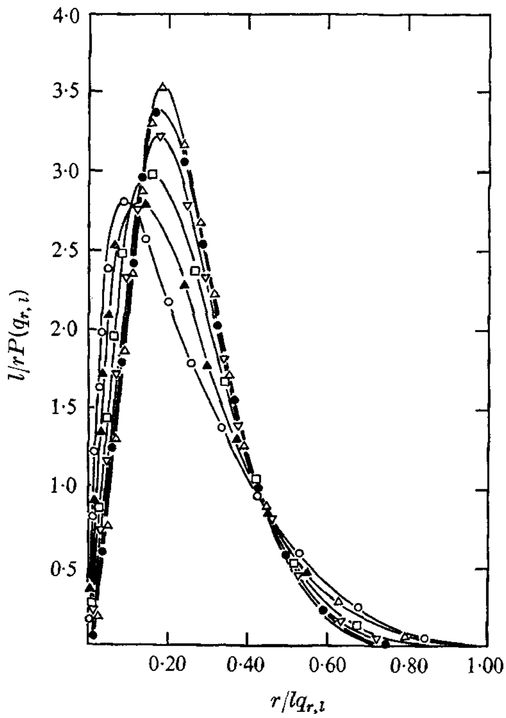

FIGURE 7. Probability density of $q_{r,l}$ for various values of l for fixed $l/r = 4$. ○, $l/\eta = 72{\cdot}5$; ▲, 145; □, 290; ▽, 580; ●, 1160; △, 2320.

for the value of $l/r = 2$. According to Onsager (1949) and Novikov (1971), this value may have special significance for the spectral energy transfer process. They argue that, in view of the quadratic nonlinearity of the Navier–Stokes equations and resulting convolution of Fourier components of the velocity, the energy tends to distribute itself across the spectrum in a cascade mode with a decrease in scale by a factor of two at each step. However, there appears to be no rigorous argument to support this idea. The increase in the values of the higher moments for $l/r = 2$ and decreasing l noted in figures 2–4 is due mainly to the increase in $P(q_{r,l})$ in the tail of the probability density for large $q_{r,l}$ as l decreases (i.e. to an increase in the relative number of large values of $q_{r,l}$).

As l/r increased ($l/r \geqslant 4$), the probability densities for different values of l became more similar, both in magnitude and shape. For fixed l/r, as l increased, the magnitude of the peak in the probability density increased and the peak shifted to larger values of $q_{r,l}$, while the tail of the density for large values of $q_{r,l}$ decreased slowly but monotonically in magnitude as l increased. As illustrated in the example given in figure 7, these changes were often so small, especially in the tail of the density for large $q_{r,l}$, that they could not be readily discerned on linear plots of the densities, although the behaviour was clear from the tabulated data. In terms of the normalized variables used in the figures, the magnitudes of these changes were roughly the same for all values of l. To examine more closely the behaviour in the tail for large $q_{r,l}$, which gives the main contribution to the

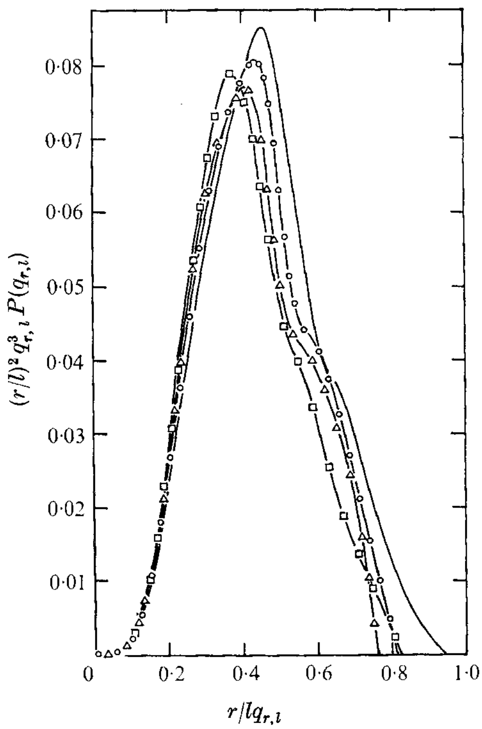

FIGURE 8. Probability densities of third moments of $q_{r,l}$ for $l/r = 4$. —, $l/\eta = 290$; ○, 580; △, 1160; □, 2320.

moments, plots of $(l/r)^{1-p} q^p_{r,l} P(q_{r,l})$ against $r/lq_{r,l}$ with $p = 2$, 3 and 4, which emphasize the behaviour in the tail, were useful. The example of such a comparison shown in figure 8 clearly shows the decrease in the value of the probability density for large $q_{r,l}$ as l increases. This behaviour is also consistent with the increase in the values of a_p for fixed l/r and decreasing l in figures 2–4. For these cases, however, the main contribution to the changes comes only from the tail of the distribution of large values of $q_{r,l}$ (i.e. from an increase in the relative number of very large values of $q_{r,l}$).

These same data were also plotted in the form of $\zeta = \ln q_{r,l}$ against $P(\xi)$, where $\xi = (\zeta - \langle\zeta\rangle)/\sigma_\xi$ and σ_ξ is the variance of ζ, to compare them with the data of Gibson & Masiello (1972). The data for their calculations, which were performed for the case $l/r = 2$ only, were obtained at a height of $z = 30$ m in the atmospheric boundary layer over the ocean. As shown in figure 9, when plotted this way the systematic changes in the tails and near the peak become much less pronounced, and the direction of the trends with changing l are reversed from those in the previous figures because of the transformation of variables. This behaviour can also be detected in the data of Gibson & Masiello, with the exception of that for their smallest l, which does not follow the trend. Otherwise, there is fairly good agreement with the present data.

If all the probability densities were identical for a given value of l/r, then according to Yaglom's analysis the value of $\mu = \mu_2(0)$ would be constant and

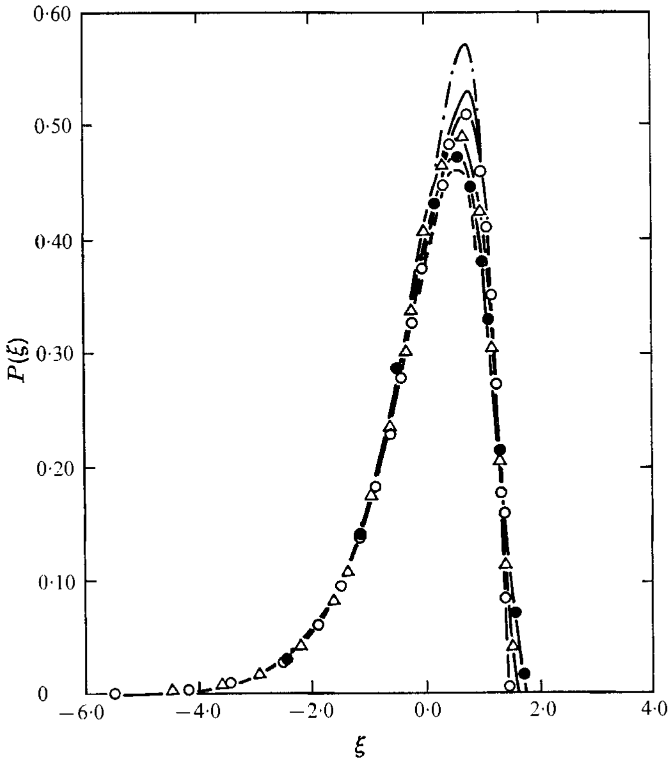

FIGURE 9. Probability density of $\xi = (\zeta - \langle\zeta\rangle)/\sigma_\zeta$, where $\zeta = \ln q_{r,l}$, and $l/r = 2$. —, $l/\eta = 36·2$; ○, 72·5; △, 145; ●, 290. ---, Gibson & Masiello, $l = 16$ cm; — · —, Gibson & Masiello, $l = 4$ cm.

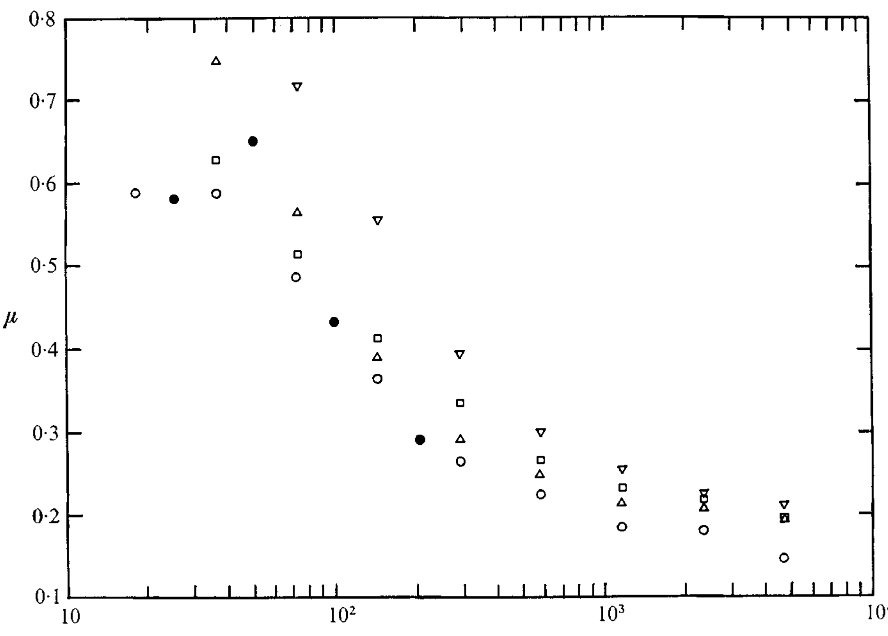

FIGURE 10. Computed values of $\mu = \sigma^2_{\ln q_{r,l}}/\ln(l/r)$. ○, $l/r = 2$; □, 4; △, 8; ▽, 16; ●, Gibson & Masiello, $l/r = 2$.

given by $\mu = \sigma_{\ln q_{r,l}}/\ln(l/r)$. Not surprisingly, the values of μ computed in this way from the present data and shown in figure 10 are a strong function of l, varying from 0·6 to 0·15 and decreasing very rapidly for the smaller values of l as l increases. The computed value of μ also depends strongly on the value of l/r, but for each fixed l/r the same trend with l is observed. As shown in figure 10, the data of Gibson & Masiello show the same trend, again with the exception of that for the smallest value of l. The rapid variation of the computed μ with l for the smallest values of l explains the origin of the widely scattered values of μ obtained by Gibson & Masiello and shows that the apparent agreement of their average value of $\mu = 0{\cdot}49 \pm 0{\cdot}2$ with values of $\mu \simeq 0{\cdot}5$ obtained from spectra of $(\partial u/\partial t)^2$, and by other means, is misleading.

The fact that the computed μ is a strong function of both l/r and l is evidence that the probability densities of the $q_{r,l}$ for different values of j are far from being similar enough to even approximately satisfy Yaglom's assumption (discussed in §2) that the $q_{r,l}$ are identically distributed for different values of j.

The second condition of scale similarity requires that two sequential breakdown coefficients $q_{r,\rho}$ and $q_{\rho,l}$ having the same value of h be statistically independent. For two statistically independent variables the joint probability density $P(q_{r,\rho}, q_{\rho,l})$ is equal to the product of the individual probability densities, or $P(q_{r,\rho}, q_{\rho,l}) = P(q_{r,\rho})\,P(q_{\rho,l})$. The moments then satisfy the relation

$$\langle q^m_{r,\rho}\, q^n_{\rho,l}\rangle = \langle q^m_{r,\rho}\rangle \langle q^n_{\rho,l}\rangle,$$

or, for $m = n = p$, $a_p(r,l) = \langle q^p_{r,l}\rangle = a_p(r,\rho)\,a_p(\rho,l)$. Thus, a necessary condition for statistical independence is that the ratio $f_p = a_p(r,l)/a_p(r,\rho)\,a_p(\rho,l)$ be equal to one for all p. Moments up to fourth order were calculated to test whether this condition was satisfied. As shown in figures 11–13, the value of f_1 is fairly close to one, ranging from 1·01 to 1·03 over the complete range of averaging lengths. This behaviour, along with the small values of the correlation coefficient described below, suggests that to this order $q_{r,\rho}$ and $q_{\rho,l}$ are nearly statistically independent. However, this test of the moments apparently becomes more sensitive for the higher moments, which exhibit strong systematic variations with l. The magnitude of variation of f_p for the higher moments for fixed l/r is a monotonically increasing function of p. As shown in figure 11, for fixed values of $l/r = 64$, $r/\eta = 72{\cdot}5$ and $l/\eta = 4640$, f_4 can become as large as $1{\cdot}4 \sim 1{\cdot}5$. The values of f_2, f_3 and f_4 reach a maximum for ρ/η equal to about 600, and decrease monotonically toward each end of the ρ interval. This behaviour suggests that $q_{r,\rho}$ and $q_{\rho,l}$ are most nearly independent when the value of ρ is close to either r or l, and that the strongest dependence occurs when ρ is roughly equal to the geometric mean of r and l $(\{(72{\cdot}5)(4640)\}^{\frac{1}{2}} = 580)$. This was also suggested by measurements of the cross-correlation coefficient R for the variables $q_{r,\rho}$ and $q_{\rho,l}$, i.e.

$$R = \langle (q_{r,\rho} - \langle q_{r,\rho}\rangle)(q_{\rho,l} - \langle q_{\rho,l}\rangle)\rangle / \langle (q_{r,\rho} - \langle q_{r,\rho}\rangle)^2\rangle^{\frac{1}{2}} \langle (q_{\rho,l} - \langle q_{\rho,l}\rangle)^2\rangle^{\frac{1}{2}}.$$

From the results in figure 11, we see that R is rather small, varying over a range from $+0{\cdot}04$ to $+0{\cdot}08$. The correlation is a strong function of ρ, exhibiting roughly the same qualitative behaviour as that of the higher moments f_p, with a maximum

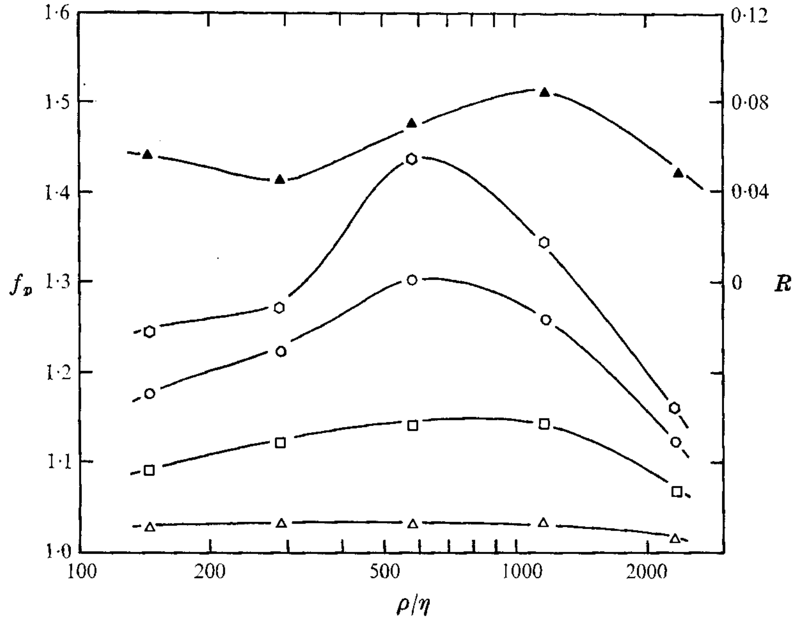

FIGURE 11. The moment ratio $f_p = \langle q^p_{r,l}\rangle / \langle q^p_{r,\rho}\rangle \langle q^p_{\rho,l}\rangle$ and corresponding correlation function R for $r/\eta = 72{\cdot}5$, $l/\eta = 4640$, and variable ρ. △, f_1; □, f_2; ○, f_3; ⬡, f_4; ▲, R.

value for intermediate values of ρ. The small values of R do not by themselves imply independence, as uncorrelated variables are not necessarily independent, but the small values of R are consistent with the fact that for first-order moments the implied degree of statistical dependence of $q_{r,\rho}$ and $q_{\rho,l}$ is very weak. However, the behaviour of the correlation coefficient and that of the higher moments for the present case appear to be related, and their dependence, however small, is accentuated in the higher-order moments.

In some other types of comparisons, such a simple correspondence did not emerge. If the ratio l/ρ is set equal to ρ/r ,then, as l/ρ increases, one would expect the correlation coefficient to decrease, since the smaller averaging segments occupy relatively less sampling volume than the larger segments. The results of such a computation for $r/\eta = 72{\cdot}5$ are shown in figure 12. R decreases smoothly from about $+0{\cdot}13$ to $+0{\cdot}062$ as l/ρ increases from 2 to 8 (a range of 64 in l/r). The corresponding f_p, however, in most cases exhibit a general increase as l/ρ increases. Similar results were found when l was varied with the values of ρ and r kept fixed. Results for $r/\eta = 145$ and $\rho/\eta = 290$ are shown in figure 13. R decreases from $0{\cdot}18$ to $0{\cdot}088$ as l/η increases from 580 to 4640. The corresponding f_p, however, change very little over the same interval.

Although no simple general relation was found between the values of the correlation coefficient and deviations of the moments from the values expected for independent probability densities, contributions to the non-self-similar behaviour of the moments from the lack of statistical independence appear to be significant, especially for the higher-order moments.

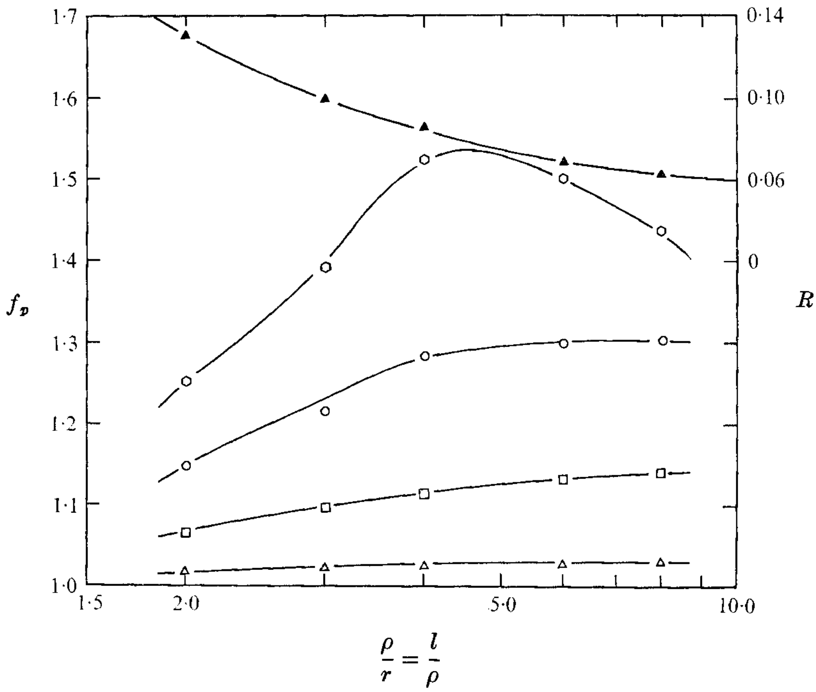

FIGURE 12. The moment ratio $f_p = \langle q^p_{r,l}\rangle / \langle q^p_{r,\rho}\rangle \langle q^p_{\rho,l}\rangle$ and correlation R as a function of ρ/r with $\rho/r = l/\rho$, and $r/\eta = 72{\cdot}5$. △, f_1; □, f_2; ○, f_3; ⬡, f_4; ▲, R.

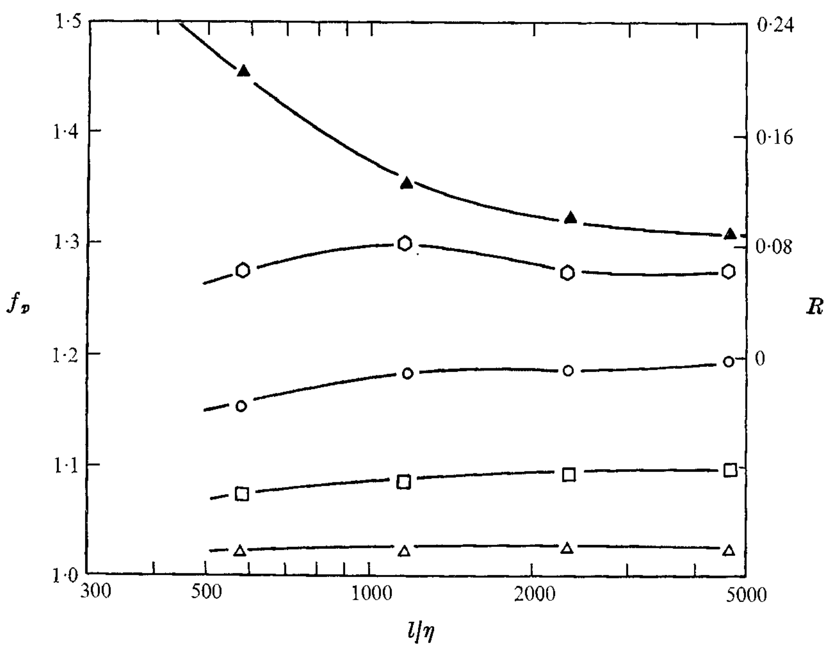

FIGURE 13. The moment ratio $f_p = \langle q^p_{r,l}\rangle / \langle q^p_{r,\rho}\rangle \langle q^p_{\rho,l}\rangle$ and correlation R as a function of l for $r/\eta = 145$, and $\rho/\eta = 290$. △, f_1; □, f_2; ○, f_3; ⬡, f_4; ▲, R.

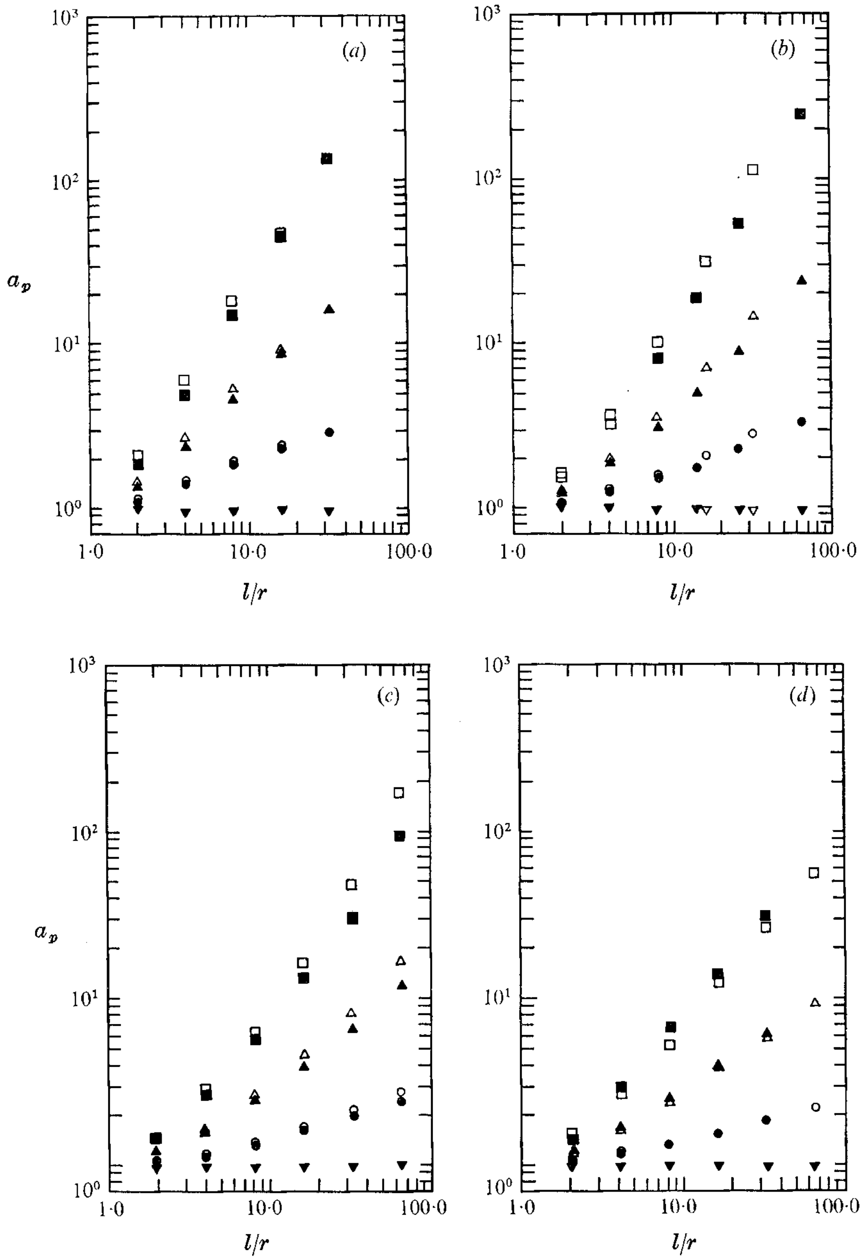

FIGURE 14. Moments of $q_{r,l}$ against l/r for fixed values of l. The data for $z = 11{\cdot}3$ m lie remarkably close to those of $z = 5{\cdot}66$ m for corresponding values of l and are not shown in the figures, except in (*d*). (*a*)–(*d*) First value of l/η: ∇, a_1; $\bigcirc$, a_2; $\triangle$, a_3; $\square$, a_4. Second value: $\blacktriangledown$, a_1; $\bullet$, a_2; $\blacktriangle$, a_3; $\blacksquare$, a_4. (*a*) $l/\eta = 72{\cdot}5$, 145; (*b*) 290, 580; (*c*) 1160, 2320; (*d*) 4640 ($z = 5{\cdot}66$ m), 5040 ($z = 11{\cdot}3$ m).

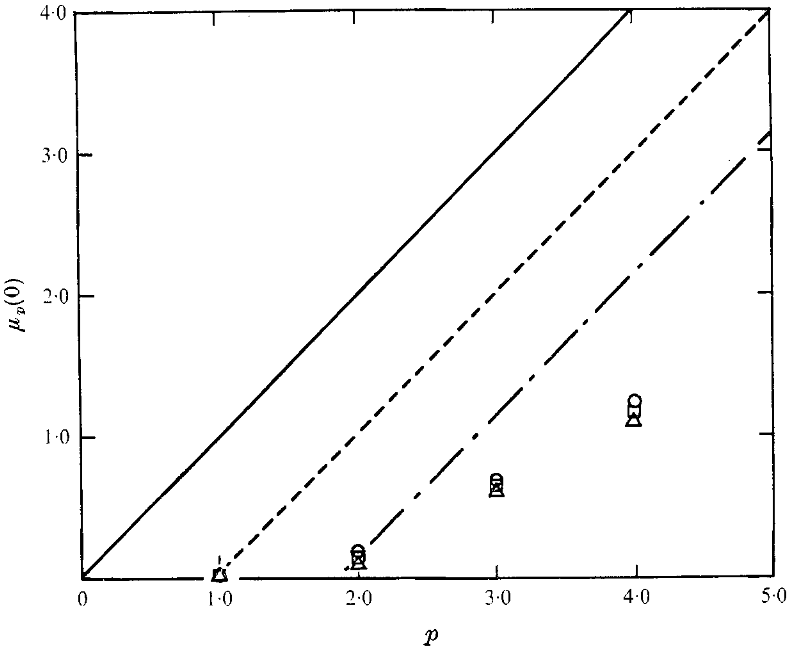

FIGURE 15. Measured values of slope of $\ln a_p$ against $\ln(l/r)$. ○, $l/\eta = 290$; □, 1160; △, 4640. Theoretical upper bounds for μ_p: ——, $\mu_p = p$; - - -, $\mu_p = p-1$; —·—·—, $\mu_p = \mu_2 + p - 2$ for $\mu_2 = 0{\cdot}15$.

Since the conditions for scale similarity are not satisfied by the present data, we cannot expect to find that the moments of the fractionation coefficient obey the simple relations which are a consequence of scale similarity. It is of interest to examine their behaviour, however, to see how it is affected by the lack of scale similarity.

The computed moments for $h = 0$ are shown as a function of l/r for fixed values of l in figure 14. From these doubly logarithmic plots we see that in most cases the moments are not the simple power-law functions of l/r required by scale similarity. For intermediate values of l, the slopes of the higher moments increase with increasing l/r, but for the smallest and largest values of l the slope (on the log plot) is fairly constant. However, even when approximate power-law behaviour is observed, the extrapolation of the slope does not pass through the point $a_p = 1$ for $l/r = 1$ as it must for scale similarity.

For corresponding values of l, the data for the two heights, $z = 5{\cdot}66$ m and $11{\cdot}3$ m, lie remarkably close together, so close, in fact, that the differences cannot be seen in the figures. Thus, although the moments are not of the form appropriate for scale similarity, they have a well-defined characteristic shape which is not a sensitive function of the height above the surface.

From figure 14 we note that for $p \geqslant 2$ the slope of the moment curves as a function of l/r decreases slowly but systematically as l increases. The behaviour reflects the decrease in the values of the moments as l increases for constant l/r noted in figures 2–4. If, despite these difficulties, the moments are fitted with straight lines with emphasis on the points for intermediate values of l/r, the resulting slopes are as shown in figure 15. The small spread in the values is caused by the systematic variation with l. The values of μ_p obtained this way increase with increasing p and are all well below the upper bounds for scale similarity given by (5) and (6), § 2. Comparing with the summary of previous measurements of μ_p given in Novikov (1971, table 1), the value of $\mu_2 = \mu \simeq 0{\cdot}15$ obtained from the breakdown coefficient is considerably smaller than values obtained from data on other statistical parameters, for which μ ranges from 0·35 to 0·51. This difference emphasizes once again the great sensitivity of the values of μ obtained to the method used to calculate it, a sensitivity noted previously in connexion with the data in figure 10. We know that it is incorrect in principle to obtain the μ_p from the moments in the present case, just as it is incorrect to obtain μ from the formula $\mu = \sigma^2_{\ln(q_{r,l})}/\ln(l/r)$, because the fundamental assumptions underlying both (4) and the relation $\mu = \sigma^2_{\ln(q_{r,l})}/\ln(l/r)$ are not satisfied. In the present case it is clear that we should not place great significance on the values obtained. The case for or against the validity of methods for deriving values of μ from other measurements is not as clear. For example, the assumptions underlying the determination of μ from spectra of ϵ are based on assumptions about the probability density of ϵ itself, but ϵ cannot be measured. Even if we assume that the hypotheses about ϵ are satisfied, the spectrum of ϵ cannot be measured either, and it is finally necessary to make the additional assumption that spectra of quantities like $(\partial u/\partial x)^2$ can be used.

As noted by Novikov, no other suitable data are available to compare with the values of μ_p for $p > 2$. Lacking suitable data, Novikov tentatively compared his results with values of the exponents from power laws fitted to the moments of squared velocity derivatives averaged over various lengths (the quantities $y_r(x)$) measured by Kholmyansky (1970), and with values deduced from higher moments of unaveraged velocity derivatives measured by Stewart *et al.* (1970). However, Novikov notes that the scale similarity theory does not apply to the moments measured by Kholmyansky and by Stewart *et al.*, remarking that, although the latter data could be interpreted in terms of the breakdown coefficient using the transition to the limit $l \to \infty$, this would violate the restricted range of scale similarity. The values of μ_3 and μ_4 deduced from these data are roughly 30–50 % larger than those obtained from the breakdown coefficient.

5. Conclusions

The assumptions and predictions of the hypothesis of scale similarity do not adequately describe or predict the statistical characteristics of the fractionation coefficient $q_{r,l}$ of the square of the streamwise velocity derivative measured in an atmospheric boundary layer. Systematic variations in the measured probability densities and consistent variations in the measured moments show that the assumption, that the probability density of the fractionation coefficient is a function only of the scale ratio, is not satisfied. The small positive correlation between adjoint values of $q_{r,l}$ and measurements of higher moments indicate that the assumption that the probability densities for adjoint values of $q_{r,l}$ are statistically independent is also not satisfied. The moments of $q_{r,l}$ do not have the simple power-law character that is a consequence of scale similarity.

As the scale ratio l/r changes, the probability density of $q_{r,l}$ evolves from a sharply peaked, highly negatively skewed density for large values of the scale ratio to a very symmetrical distribution when the scale ratio is equal to two, and then to a highly positively skewed density as the scale ratio approaches one. There is a considerable effect of heterogeneity on the values of the higher moments, and a small but measurable effect on the lowest moment (mean value). The moments are roughly symmetrical functions of the displacement of the shorter segment from the centre of the larger one, with a minimum value when the shorter segment is centrally located within the larger one.

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